

## Multiple Choice:

4) A

5) C

8) B

10) C

15) C

16) B

18) A

22) C

23) D

24) D

27) C

35) B

36) D

37) C

38) C

## 4.5- Frequency Tables

	<sup>D</sup> Drink	<sup>DD</sup> Don't Drink	
<sup>Sm</sup> Smoke	315	165	480
<sup>DS</sup> Don't smoke	585	135	720
	900	300	1200

Find the probability that:

a) the student smokes  $P(S_m) = \frac{480}{1200} =$

b) the student smokes given that he is a drinker

$$P(S_m | D) = \frac{315}{900}$$

c) the student drinks and smokes

$$P(D \cap S_m) = \frac{315}{1200}$$

d) the student smokes or he doesn't drink

$$P(S_m \cup DD) = \frac{(315 + 165 + 135)}{1200}$$

2) a)  $P(N) = 1800/2000 = 90\%$

b)  $P(H|L) = 10/100 = 10\%$

c)  $P(N \cup R) = (190+1710+90)/2000 = 99.5\%$

3) a)  $P(<35) = 42968/151616 = 0.283$

b)  $P(C) = 30781/151616 = 0.203$

c)  $P(<35 \cap C) = 10174/151616 = 0.067$

d)  $P(C|<35) = 10174/42968 = 0.237$

e)  $P(<35 \cup C) = (32794+10174+20607)/151616 = 0.419$

## BAYES' RULE (aka Multistage Problems)

- Let A = 1<sup>st</sup> event
- Let B = 2<sup>nd</sup> event

We know that  $P(B|A) \neq P(A|B)$

So how can we find  $P(A|B)$ ? In other words, how can we find the probability that the first event (A) happened given that we know the second event (B) happened?

Bayes' Rule: (page 356)

$P(A|B) =$  \_\_\_\_\_

However, this rule is long, complicated, and a very difficult formula.  
So instead, we use **TREE DIAGRAMS**

Think of this example:

A basketball player shoots 2 free throws. The following probabilities apply:

Prob. of making the 1<sup>st</sup> = 0.6

$$P(X_1) = 0.60$$

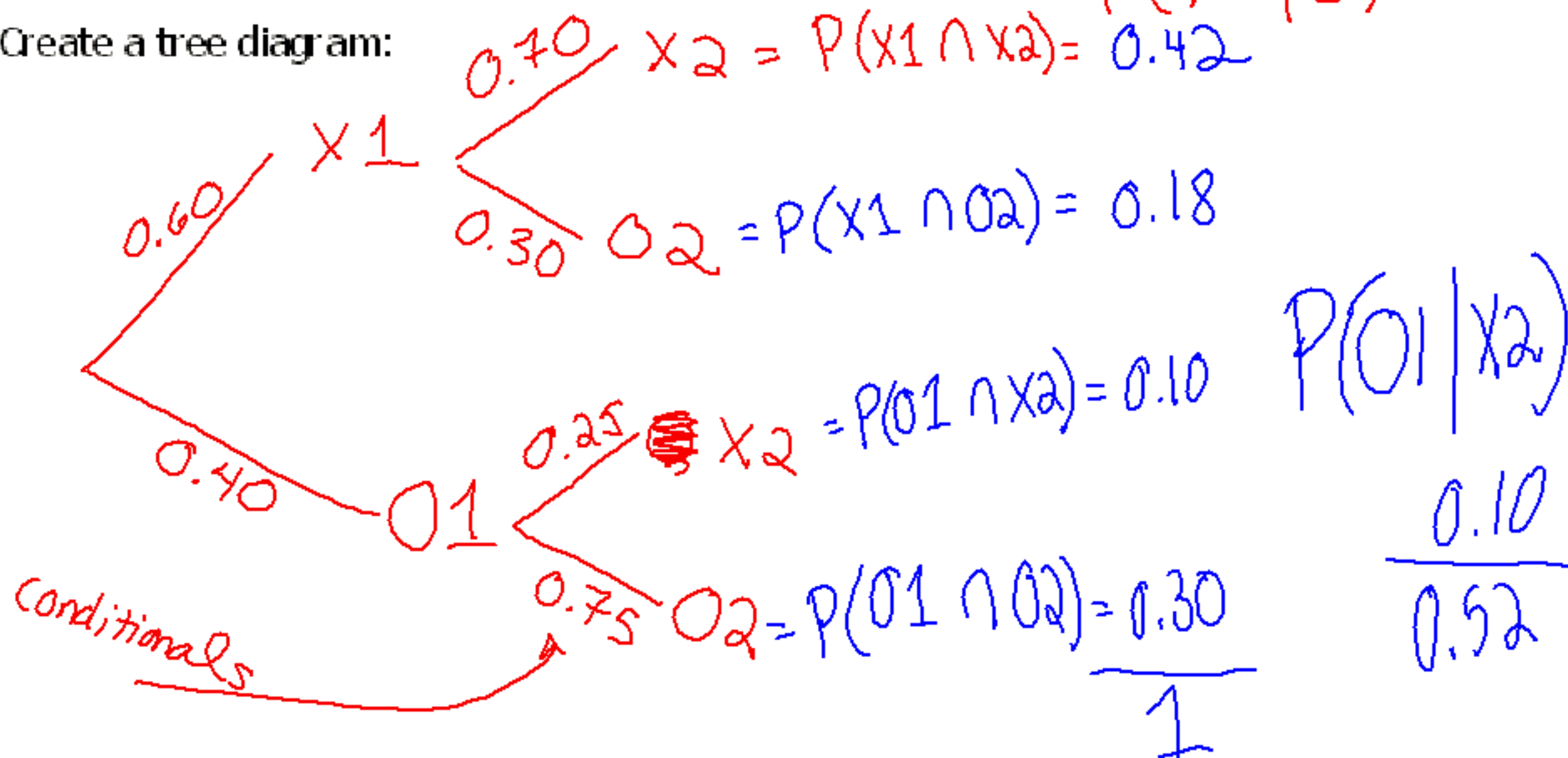
$$P(X_2 | X_1) = 0.70$$

Prob. of making the 2<sup>nd</sup> **given that** you make the 1<sup>st</sup> = 0.70

Prob. of making the 2<sup>nd</sup> **given that** you missed the 1<sup>st</sup> = 0.25

$$P(X_2 | O_1) = 0.25$$

Create a tree diagram:



So now lets answer some questions:

1. What is the probability of missing the 1<sup>st</sup> free throw?  $P(O1) = 0.40$

2. What is probability of making the first free throw **and** making the second free throw?

$$P(X1 \cap X2) = 0.42$$

3. What is the probability of making the second **given that** you missed the first?

$$P(X2|O1) = 0.25$$

4. What is the probability of missing the second **given that** you made the first?

$$P(O2|X1) = 0.30$$

Now lets try some harder ones: (use the probability rules!!)

1. What is the probability of making the second free throw?

$$P(X_2) = 0.42 + 0.10 = 0.52$$

2. What is the probability of missing the second free throw?

$$P(O_2) = 0.30 + 0.18 = 0.48$$

So lets say you went to the bathroom and didn't see the first free throw.....

3. What is the probability you made the first given that you make the second?

$$P(X_1|X_2) = 0.42/0.52 = 0.808$$

4. What's the probability you missed the first given that you make the second?

$$P(O_1|X_2) = 0.1/0.52 = 0.192$$

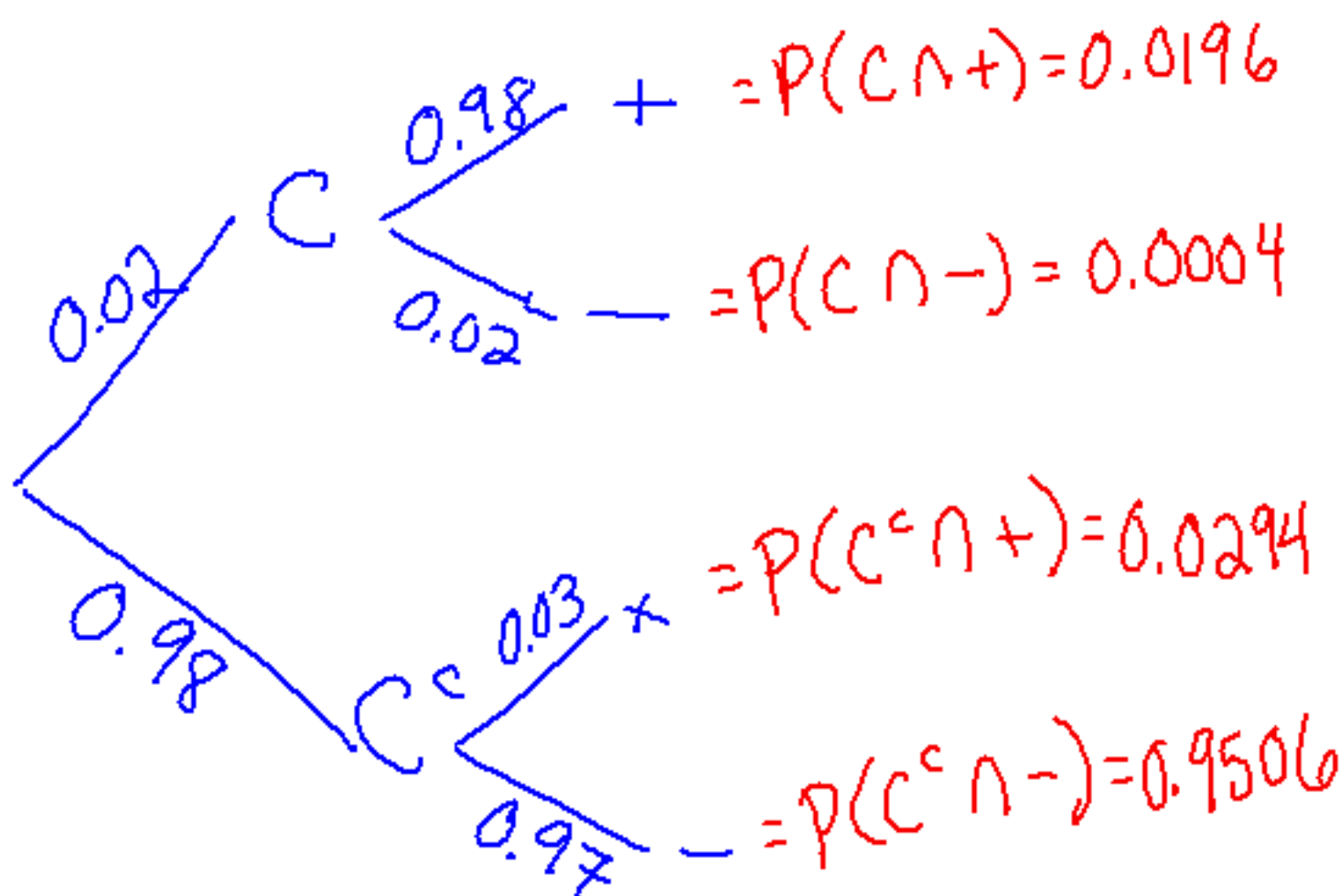
# Example #1

- A cancer clinic gives free cancer test (hypothetically)
- It is known that 2% of the people that come into the clinic have cancer (hypothetically)  $P(C) = 0.02$
- It is known the test comes up positive in 98% of people with cancer  $P(+|C) = 0.98$
- It is known the test comes up positive in 3% of people without cancer  $P(+|C^c) = 0.03$

Create the tree diagram:

1<sup>st</sup> event = cancer  
2<sup>nd</sup> event = result of test

$$8) P(C|-) = 0.000421$$



$$1) P(+|C) = 0.98$$

$$2) P(+|C^c) = 0.03$$

$$3) P(-|C) = 0.02$$

$$4) P(-|C^c) = 0.97$$

$$5) P(+) = 0.049$$

$$P(-) = 0.951$$

$$6) P(C|+) = \frac{0.0196}{0.049}$$

$$7) P(C^c|+) = \frac{0.0294}{0.049}$$



## MC answers (Ch.4A warm ups)

1) B

2) A

6) C

7) C

9) B

11) A

12) C

13) D

17) B

19) B

20) D

21) D

25) B

26) B

32) C

33) C

38) C

39) D

**Example #2:**

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

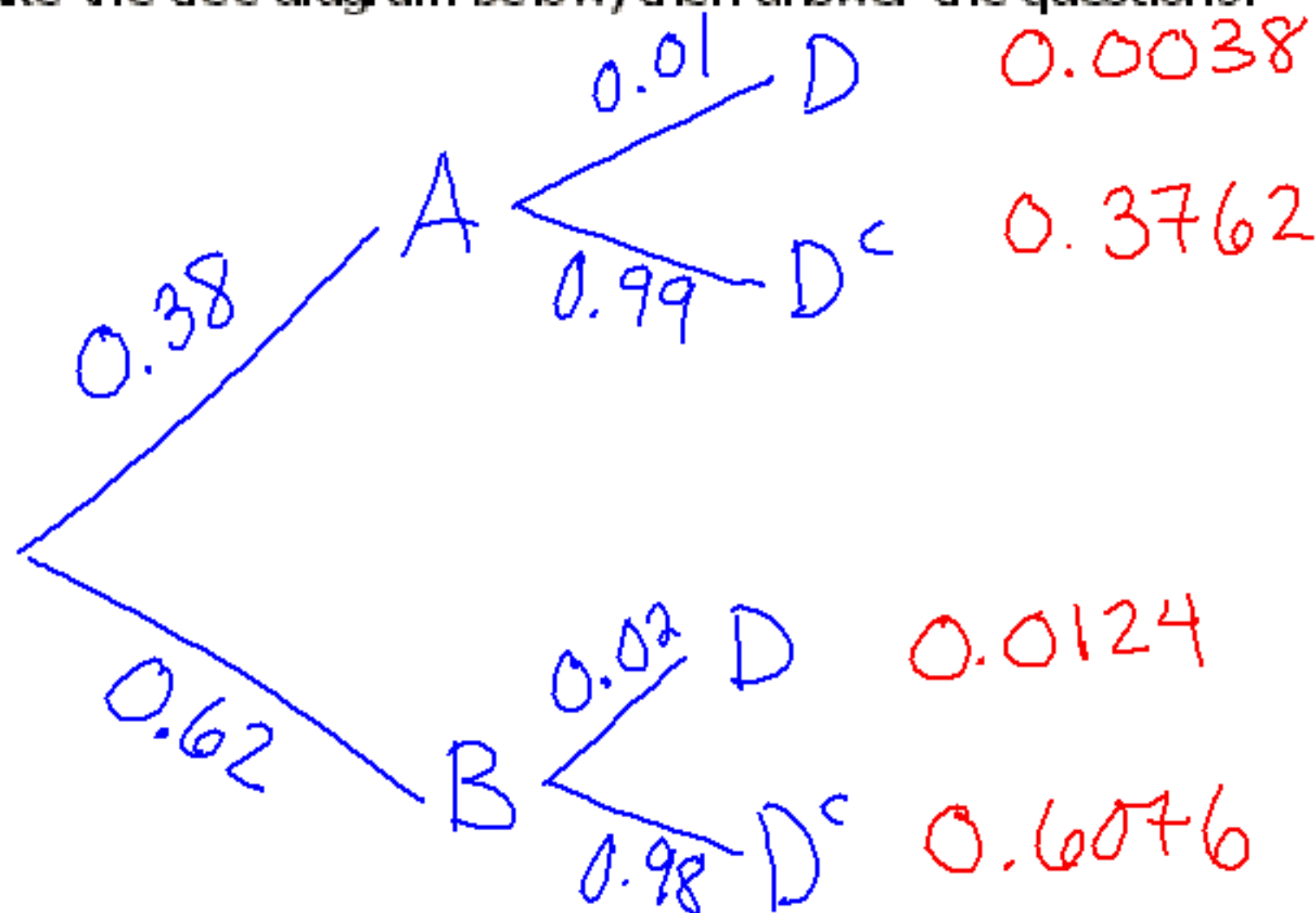
$$P(D|A) = 0.01$$

$$P(D|B) = 0.02$$

$$P(A) = 0.38$$

$$P(B) = 0.62$$

Make the tree diagram below, then answer the questions:



$$* 1) P(D^c) = 0.3762 + 0.6076 = 0.9838$$

$$* 2) P(B | D^c) = \frac{0.6076}{0.9838} = 0.6176$$

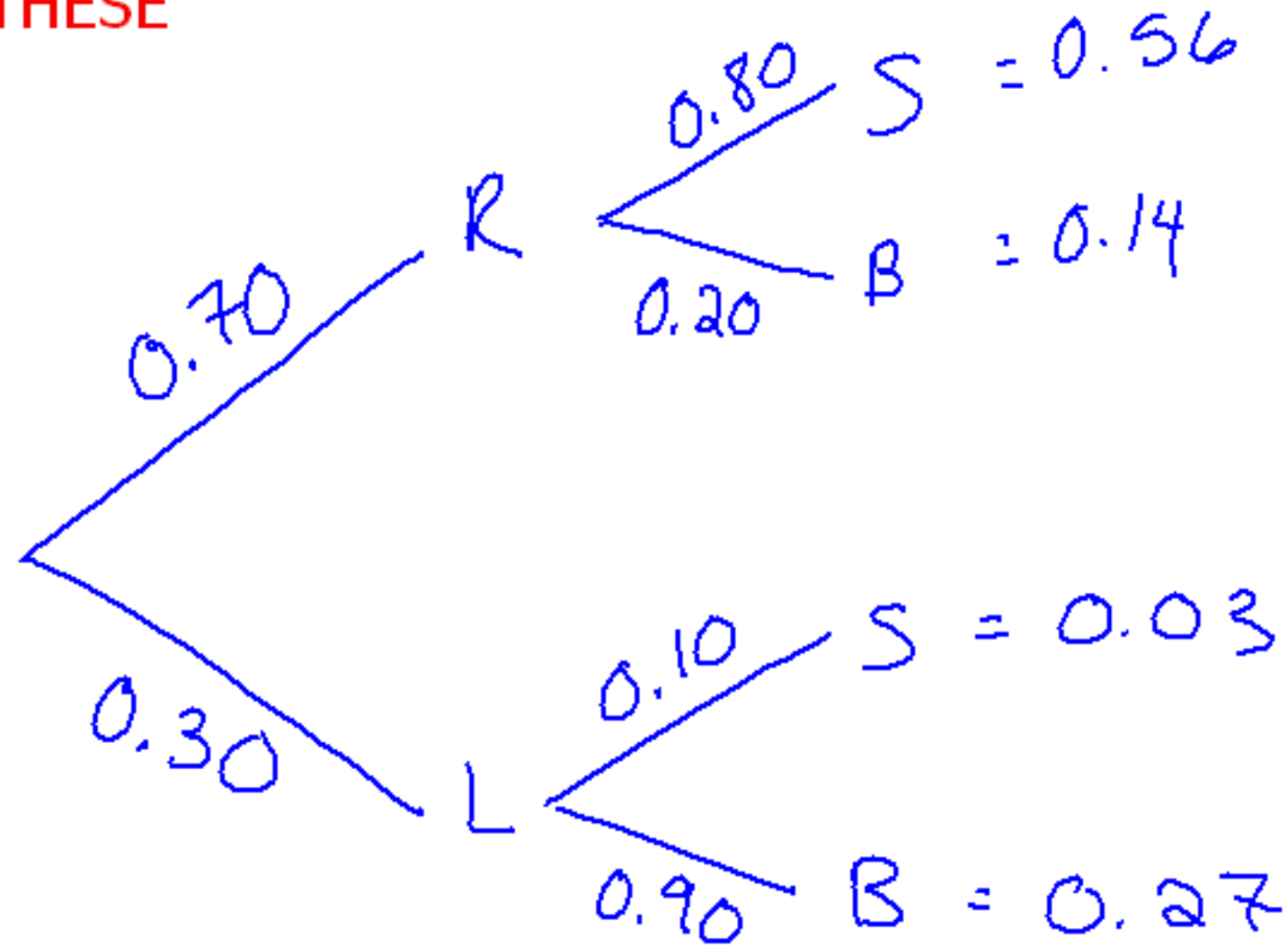
$$3) P(A | D) = \frac{0.0038}{0.0162} = 0.2346$$

$$P(B | D) = \frac{0.0124}{0.0162} = 0.7654$$

\* Do #2 on next page

\* Then do # 1 & 3 on worksheet 4.5

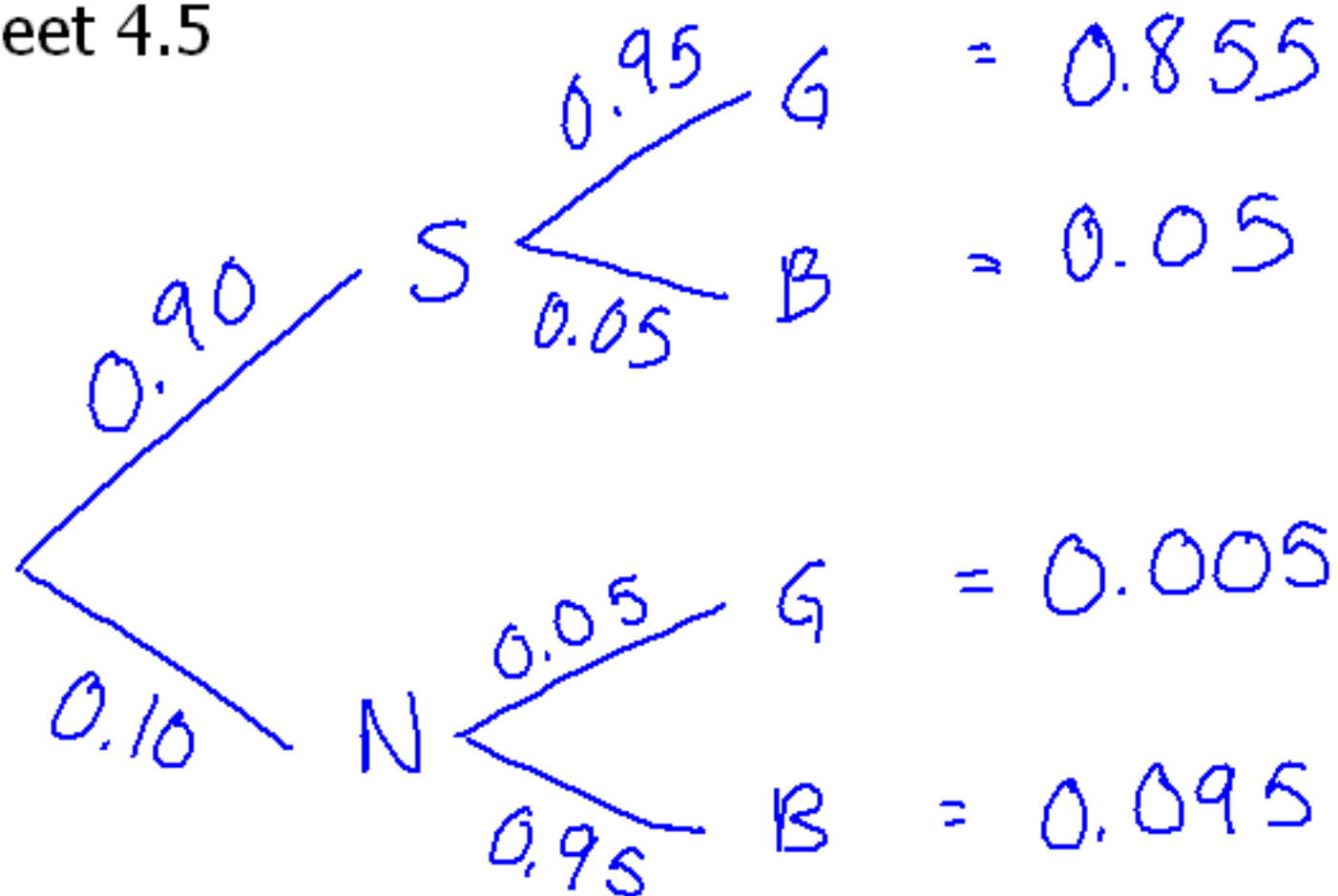
TRY THESE  
#2)



$$P(L|B) = 0.27 / 0.41 = 0.659$$

# Worksheet 4.5

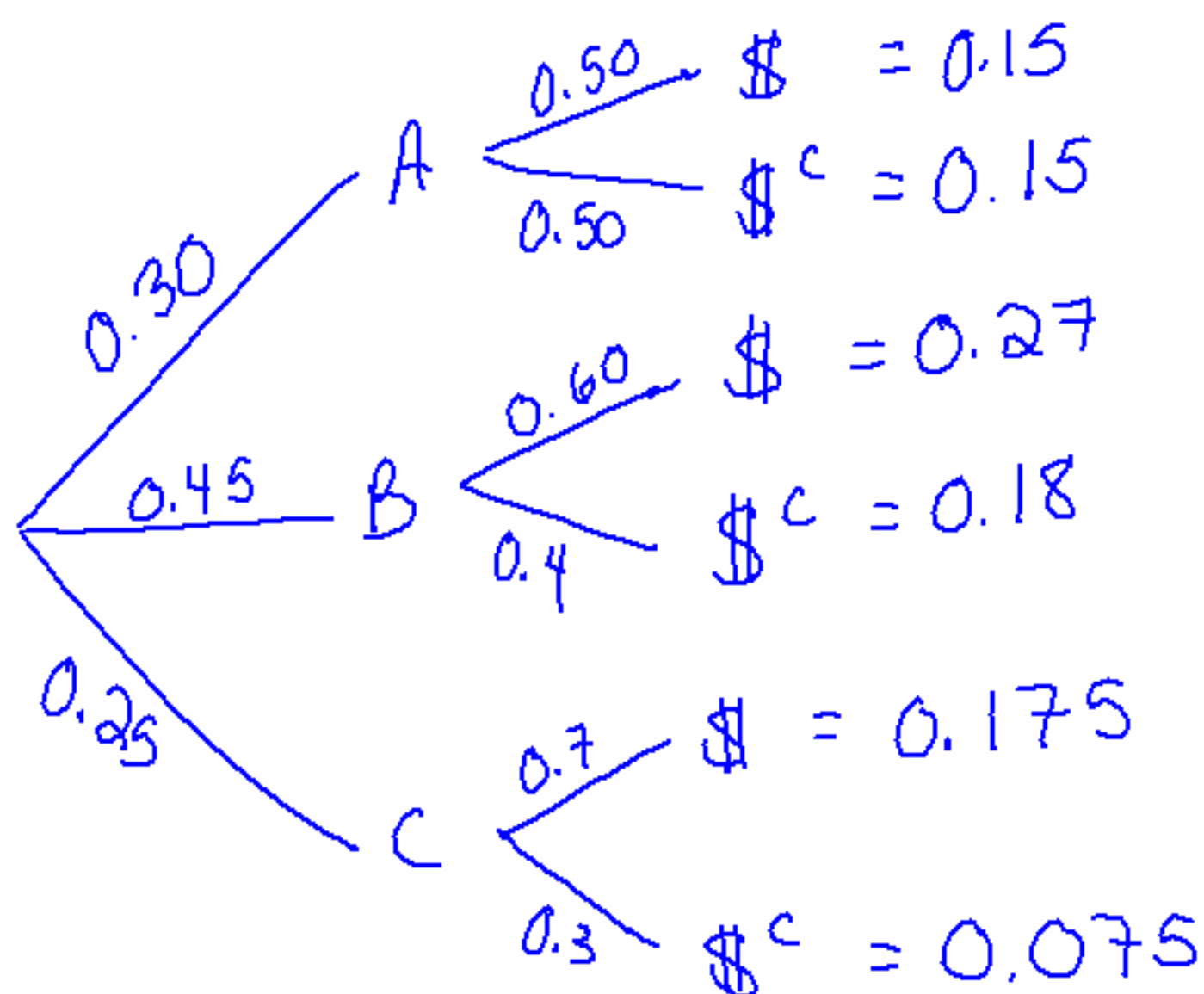
1)



a)  $P(G) = 0.855 + 0.005 = 0.86$

b)  $P(S|G) = 0.855/0.86 = 0.994$

3)



a)  $P(\$) = 0.15 + 0.27 + 0.175 = 0.595$

b)  $P(A|\$) = 0.15/0.595 = 0.252$

# PERMUTATIONS AND COMBINATIONS

## Permutations:

prizes, positions

\* Arranging...  $n$  objects, taking  $r$  at a time

\* ~~Formula:~~

On calculator: total  
type  $n$ ,  
then MATH --> PRB -->  $nPr$  --> ENTER  
then type  $r$  — # choosing

\* Order DOES MATTER

$ABC \neq BAC$

\* More possibilities

\* Example: For a fundraiser, 50 raffle tickets are sold. 1st, 2nd, 3rd place prizes will be awarded. How many different ways are there to hand out these prizes?

$$50 \text{ } nPr \text{ } 3 = 117,600$$



## Combinations:

Committee

\* The number... of groups of  $n$  objects, picked  $r$  at a time  
~~sample~~ \* sample

\* Formula:

On calculator:

MATH --> PRB -->  $nCr$

\* Order DOESN'T matter

\* Less... possibilities

$ABC = BAC = CAB = \dots$

\* Example: We want to randomly select a sample of 8 students from a class of 21. How many different samples are possible?

$$21 nCr 8 = 203,490$$

## Counting Rules:

- \* Looking for... **all possible ways to combine different groups**
- \* For groups of sizes  $m$ ,  $n$ , and  $t$  (using all possibilities in each group)
- \* Formula:  **$m \cdot n \cdot t$**

*\* not choosing*

- \* Example:

A person wants to make a sandwich from a buffet. On the buffet, there are 3 types of bread, 4 types of cheese, 3 types of lunchmeat, and 3 different condiments. If the person wants a sandwich that consists of bread, cheese, lunchmeat, and a condiment, how many different sandwiches can he make?

$$3 \cdot 4 \cdot 3 \cdot 3 = 108$$

**Putting it together...**

$$(30 \text{ nPr } 2) \cdot (28 \text{ nCr } 6)$$

$$327,763,800$$

Wksht:

or  $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$1) 7 nPr 7 = 5040$$

$$2) 12 nPr 10 = 239,500,800$$

$$3) \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{26} \cdot \underline{25} = 65,000,000$$

$$4) 10 nCr 5 = 252$$

$$5) 5 nPr 5 = 120 \times 2 = 240$$

$$6) (3 nCr 2) \cdot (7 nCr 2) = 63$$

$$7) 13 nPr 4 = 17160$$

$$11) 13 nCr 4 = 715$$